UNIVERSITY OF CAMBRIDGE SUMMER PROGRAMME 2016

AT THE CANADIAN INTERNATIONAL SCHOOL OF HONG KONG

INTRODUCTION TO SUSTAINABLE ENERGY

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1 About this course

Our modern standard of living is sustained by large quantities of energy. At the moment we get most of this energy by burning fossil fuels, but this is manifestly unsustainable: we are burning fossil fuels at a far higher rate than they are being produced, so at some point they will run out. The burning of fossil fuels is also causing troubling changes to the Earth's climate. In this course we review the basic science of climate change and ask what a world powered by sustainable energy would look like? How much energy can we hope to produce from different sustainable sources? How much energy do we need to sustain our standards of living? We will estimate these quantities and build a quantitative picture of the options available to us. This course draws heavily on the wonderful book "sustainable energy without the hot air" by Prof David MacKay FRS of Cambridge University. The book, which contains much more content that this course, can be bought in paper form or downloaded free as a pdf from `www.withouthotair.com`. I would urge anyone who enjoys the course to do so.

This course has three main objectives. The first, as advertised, is to learn about sustainable energy. However, understanding sustainable energy requires broad range of fundamental physics ideas, and learning this physics is a second central aspect of this course. Finally, the engineering of sustainable energy is in reality a very complicated technical business, but we will make progress by estimation. Being able to estimate how big various effects are is often the first step towards a full understanding of a complex phenomena and the hallmark of a good physicist. Practicing estimation is the third objective of this course.

2 Quantities every physicist should know

Just as to understand the size and nature of numbers one needs to be fluent with multiplication tables, so to understand the size and nature of effects around us, one needs to know a few facts about the world. We list here a few that all Physicists should be familiar with. If you feel there are some missing from the list that you have found useful at times, please suggest them.

Physical Quantities

quantity	**magnitude**	**unit**	**comment**
Seconds per year	$\pi \times 10^7$	-	-
Distance to Sun	8.3	light minutes	150 million km
Distance to Moon	1.3	light second	384,400 km
Radius of Earth	6.4×10^3	km	-
Mass Earth	6×10^{24}	kg	
Density water	10^3	kg/m^3	-
Density air	1.3	kg/m^3	STP
Avogadro's number	6.03×10^{23}	-	-
Boltzmann's constant	1.38×10^{-23}	J/K	
Speed of light	3×10^8	m/sec	-
Speed of sound	3.3×10^2	m/sec	RTP
Planck's constant / 2π	1.05×10^{-34}	J.sec	(or without /(2π))
Charge on electron	1.6×10^{-19}	Coulomb	-
Mass of proton	1.67×10^{-27}	kg	-
Wavelength of visible light	$700 - 400$	nm	red laser 630nm
Distance to closest star	4.2	light years	Alpha Centauri
Temperature of surface of Sun	6000	K	-
Solar flux incident on Earth	1.4	Kw/m^2	summer, tropics, no clouds
Acceleration due to gravity	9.81	m/sec^2	on surface of Earth
RTP	293, 10^5	K, Pa	Pa = 1 N/m^2
Specific heat water	4.12	J/(gm K)	≡1 cal/(gm C) - high

3 Energy and Power

We are familiar with many different types of energy. A moving object has kinetic energy, a hot object has thermal energy, a ball on top of a hill has gravitational potential energy and a can of petrol contains a good deal of chemical energy. However, none of this helps us much with the question "what is energy?". The truth is that we don't have a good answer to this question, the best that can be said is that energy is a quantity that can be transferred between objects and between different forms, but never created or destroyed. However, moving energy from the forms and places we find it to useful forms in useful places — for example by burning petrol in a car to turn chemical energy into kinetic energy and hence making a journey — underpins life and civilization. In this course we will study many different forms of energy, where they are found and how we might use them.

3.1 Aside on units

In this course we will measure energy in Joules. In the SI system of units, a Joule is not a fundamental unit, but rather the energy expending pushing a force of 1 Newton through a distance of 1 meter, using the classic mechanics formula Work Done = Force × Distance, so 1J=1Nm. A Newton is also not a fundamental unit, but rather specified by $f = ma$ as a mass multiplied by an acceleration, with units of kg m s^{-2}, so the Joule can be written in fundamental SI units as 1J=1kg m^2 s^{-2}

3.2 Some forms of energy

1. Heat energy: It requires an energy $E = C\Delta T$ to raise an objects temperature by ΔT, where C is the objects heat capacity. The heat capacity of an object is proportional to how much stuff it contains, so we normally quote the heat capacity of a material as the heat capacity per gram, c. The actual heat capacity of an object is the heat capacity per gram of the material the object is made from multiplied by the object's weight in grams. The most important heat capacity to know is that of water, $c_w = 4.12$ J/(gm K).

2. Kinetic Energy: An object with mass m moving with a speed v carries a kinetic energy $E = \frac{1}{2}mv^2$.

3. Gravitational potential energy: An object with mass m that is elevated by a heigh h in a gravitational field g gains a gravitational potential energy $E = mgh$.

4. Electrical energy: If a charge q is moved through a voltage V it acquires an electrical potential energy $E = qV$.

5. Light energy: Light can be thought of as a electro-magnetic waves or as made up of particles called photons. In either perspective light contains energy, but in the photon perspective this is very simple: a photon with frequency f carries an energy $E = hf$, where h in Plank's constant. This energy must be supplied to make the light and will be deposited in any object that absorbs the light.

6. Sound energy: Sound waves also carry energy.

Exercise 1. *How much kinetic energy does a moving car have? How much does a moving bullet have?*

Exercise 2. *How many Joules of energy are required to heat a kettle of water from room temperature to boiling.*

Exercise 3. *I try to boil the water in my kettle with kinetic energy by firing bullets into it. How many bullets do I require?*

Exercise 4. *I drop a stone from head height. How fast is it moving when it hits the ground.*

3.3 Power

We earlier calculated that it takes around $5 \times 10^5 J$ of heat energy to boil the water in a kettle. However, as we wait for our cup of tea, we are probably more interested in how long it will take us to boil the kettle. We know how much energy we need, but to work out how long it will take we need to know how fast we are adding energy to the water. This is what is measured by power: power is the rate at which energy is transferred either between objects or between forms, and it is measures in Joules per second (J s^{-1}), also known was Watts.

Exercise 5. *If the kettle has a power rating of 3kW, how long does it take the kettle to boil?*

4 Electro-Magnetic Radiation, AKA Light

Outer-space is very cold — about 4K or -269°C — and consequently very uninhabited. In contrast the Earth is, on average, a rather more comfortable 288K (16 °C). Why is the Earth so warm? Because it is heated by the Sun. So how does the Sun heat the Earth? We all know that when an object gets hot, it starts to emit light: this is how old-fashioned incandescent light bulbs work. The emitted light carries energy away from the hot object which consequently cools down, and will heat up anything in the vicinity that absorbs it.

All objects radiate energy in this way, but we only see objects glow when they get very hot indeed. There are two reasons for this. Firstly, the hotter an object is the more it radiates. This is quantified by the Steffan-Botlzmann law as

$$J = \sigma T^4 \tag{1}$$

where T is the object's temperature, $\sigma = 5.67 \times 10^{-8} J s^{-1} m^{-2} K^{-4}$ is the Steffan-Boltzmann constant and J the total energy, in Joules, radiated out per unit area of the body in question per second.

However, it is not just that cool objects do not radiate much light. If you heat something up you first cannot see it, then it turns red then, when it gets really hot, it turns white. The color of the emitted radiation depends on the temperature of the object. Visible light is an electro-magnetic wave, with red light having a wavelength of around 700nm and blue light having a wavelength of around 400nm. Electromagnetic waves can also be formed with other wavelengths. We don't perceive these as light, but in physics terms they are the same phenomena. At longer wavelengths than red is first infra-red radiation, which we feel as heat but can't see, then beyond that microwaves and radio-waves. At shorter wavelengths than blue is ultra-violet, which gives us sun-burn, then, at very short wavelengths, x-rays and gamma-rays. The full electro-magnetic spectrum is shown in fig. 1.

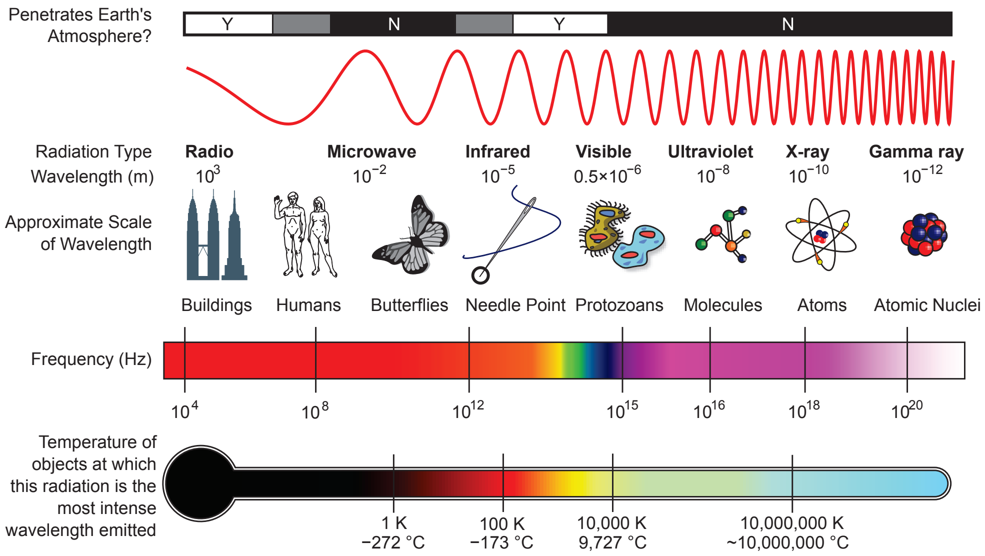

Figure 1: The electromagnetic spectrum.

Hot bodies radiate energy at a whole range of wavelengths, but the shape of the spectrum is always the same. Fig. 2 shows this spectrum — the power a body will emit at each wavelength when heated to a temperature T. The peak in this spectrum (i.e. the wavelength at which most power is emitted) is related to the temperature via the relation $\lambda_{max} = \frac{b}{T}$, where $b = 2.898...10^{-3}$ K m is known as Wien's displacement constant. Thus higher temperatures lead to emission of waves with shorter wavelengths.

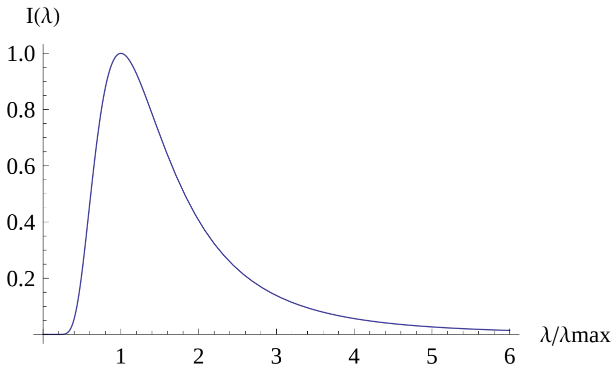

Figure 2: The power $I(\lambda)$ radiated at each wavelength by a hot body. The spectrum is always the same shape, but hotter objects radiate with shorter wavelengths with $\lambda_{max} = \frac{b}{T}$, where $b = 2.898...10^{-3}$ K m.

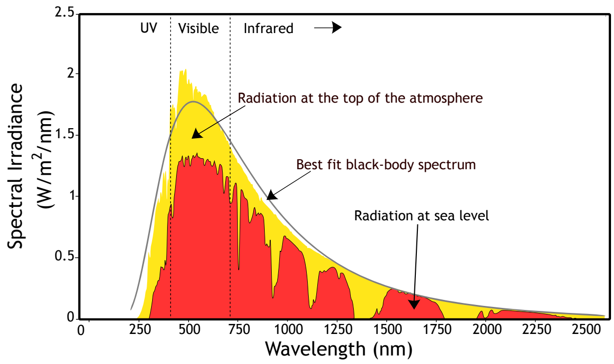

Figure 3: Spectrum of the Sun's light arriving at Earth. Yellow: Spectrum at the top of the atmosphere. Red: Spectrum at the surface of the Earth after atmospheric absorption. Black: best fit theoretical line.

Exercise 6. *What wavelength of radiation are you emitting the most of right now? Why can't I see you glowing?*

Exercise 7. *What wavelengths does the sun emit? Estimate it's temperature. Fig. 3 shows the full solar spectrum. What is the actual temperature of the sun?*

Exercise 8. *Fig. 4 shows the spectrum of the cosmic microwave background radiation that permeates all of space. What is the temperature in outer-space, far from any star or source of heat?*

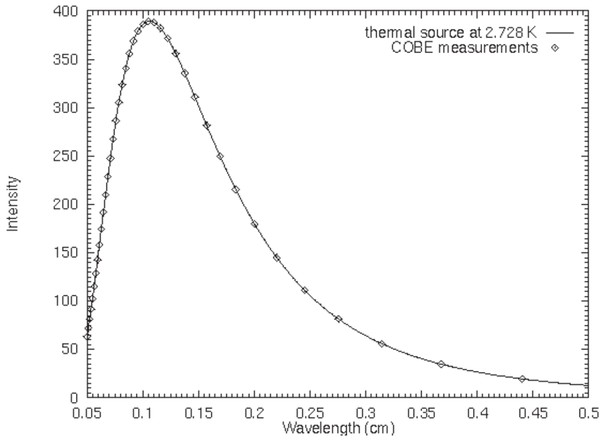

Figure 4: Spectrum of the cosmic-microwave background.

5 Unsustainable Energy and The Earth's Climate

Exercise 9. *What is the total power of the Sun in Watts? That is, how much energy does it radiate into the Solar System every second?*

Exercise 10. *What is the power of the sunlight hitting a square meter of ground in Hong-Kong?*

Exercise 11. *You just calculated the power of the sunlight hitting a square meter of ground. The square meter of ground absorbs this radiation but in turn emits its own radiation. Assuming the temperature of the ground is not changing, calculate the temperature of the ground. How does this compare to the actual temperature of the Earth? Actually the Earth reflects about 30% of the Solar radiation without absorbing it. How would this modify your prediction for the temperature of the Earth?*

The above calculations for the temperature of the Earth are surprisingly accurate, but they all predict that the Earth should be a few tens of degrees colder than it really is. This is a small difference in physics terms but a big difference in human terms: an Earth with an average temperature of $-20°C$ would not be a nice place to live. The reason the Earth is habitable is because its atmosphere acts a a greenhouse, letting the incoming solar radiation in but absorbing the infra-red radiation emitted by the earth.

Exercise 12. *(Difficult) A perfect greenhouse would let all the solar radiation through but absorb all the radiation emitted by the Earth. How much would this increase the Earth's temperature by?*

The greenhouse effect of the atmosphere is thus not a bad thing: it makes the Earth habitable. However, if we put more greenhouse gasses into the atmosphere we would expect the greenhouse effect to increase and the temperature of the Earth to increase. The two main greenhouse gasses in the Earth's atmosphere are water vapor and CO_2, which are responsible for approximately 75% and 25% of the warming effect respectively.

However, human civilization is changing the above balance. Almost all of our energy needs —- the petrol in our cars, the coal used to generate our electricity and the gas that heats our houses — are met by burning fossil fuels. These are the fossilized remains of animals and plants that lived on the Earth many years ago and they contain chemical energy that we release when we burn them. The chemical energy is in the from of chemicals called hydro-carbons, and when these are burnt they turn into water and CO_2. We are thus releasing extra CO_2 into the atmosphere, and increasing the greenhouse warming effect. In fig. 5 we see a graph of CO_2 concentration in the atmosphere as a function of time. Since the industrial revolution it has increased by about 40%.

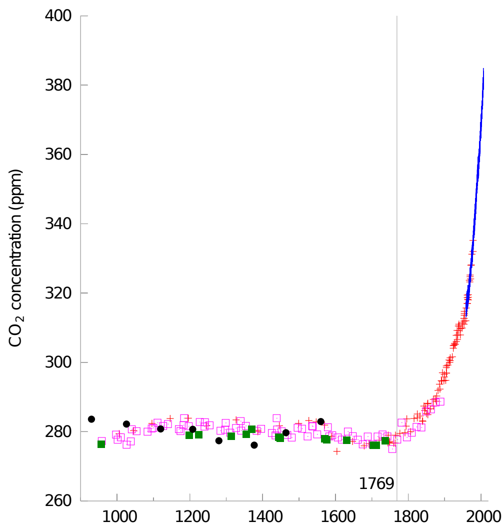

Figure 5: Atmospheric CO_2 concentration for the last 1000 years. Notice the effect of the industrial revolution.

Exercise 13. *Estimate how much of a temperature rise at 40% increase in the atmospheric concentration of CO_2 might give rise too. Should this worry us?*

Exercise 14. *Where does the chemical energy in fossil fuels come from?*

There is another problem with fossil fuels, which is that there is a finite amount of them. They were laid down over hundreds of millions of years, but we are burning them very quickly. At some point we will run out. Exactly when this is likely to happen is hotly debated, but that it must eventually happen is a clear fact. This is what we mean by an unsustainable source of energy: one which there is a finite amount of, and which will eventually run out. In contrast, there are also sources of energy in our environment that are continually renewed by the Sun, (for example the energy in incoming solar radiation) that we can use, but which will never run out. These sources of energy we call sustainable. There are thus two reasons to ask how we might power our civilization sustainably: in the long run we must because our unsustainable choices will run out, in the short term we have grounds to worry that using unsustainable sources of energy is warming the planet. This brings us to the main question of this course: how can we sustainable power our civilization?

6 The balance sheet

During this course we will build a balance sheet, comparing how much energy we need and how much sustainable energy might actually be available. The energy needs we will calculate per-person per day, assuming a person with a western lifestyle. The amount of energy available is a bit more subtle: it depends on how much space is available to our person to harvest the energy from. We will do four different calculations, one for a person in Hong-Kong, where people have on average 150m^2 each, the UK where people have on average 4000m^2 each, China, where people have on average 7000m^2 each, and the USA where people have on average 32000m^2 each. In each case we will again work out energy available per person per day. Over the course, we will populate the following tables and see whether a sustainable future is a possibility.

Energy Source	**Hong-Kong**	**UK**	**China**	**USA**
Solar				
Wind				
Geothermal				
Wave				
Tide				
Hydroelectric				
Nuclear Fision				
Nuclear Fusion				

Energy Need	**Energy per person per day**
Cars	
Flights	
Lighting	
Heating and Cooling	
Gadgets	
Food	
Making Stuff	
Moving Stuff	

7 Cars and Planes

We use a lot of energy transporting ourselves, but how much?

7.1 Cars

We mostly transport ourselves in cars. How much energy is required to power the average westerner's car habit? We can start by estimating this by using experience from our own daily lives.

Exercise 15. *How far does a typical person drive each day? How much fuel does this require each day? How much energy is this per day? Congratulations, this is your first estimate. Put it into the table above.*

Where does this energy go? There are two main answers to this question. Firstly, every time I set the car moving the engine turns fuel energy into the car's kinetic energy, which is later lost as heat when I break. Secondly, as the car goes along, it must move the air in-front of it out of the way by setting it in motion, so, as sketched in fig. 6, the car is continually giving kinetic energy to the air it is traveling through. If the car has frontal area A and moves at a speed v then in a time t it sets in motion a volume of air Avt out of the way. This is a mass of air given by ρAvt. where ρ is the density of air. The air must be accelerated to roughly the same speed as the car is moving, so it acquires a kinetic energy $E \sim \frac{1}{2}\rho Atv^3$. Dividing this by the distance the car has traveled vt, gives us an energy consumption per meter of $\frac{1}{2}\rho Av^2$.

Exercise 16. *Assuming we drive our 50km by driving for an half an hour at 100km/hour, use the above energy consumption per meter to estimate how much energy the car would need every day. If the car engine in only $\frac{1}{4}$ efficient at turning the energy in the fuel into mechanical energy, how much energy would the car use every day?*

The above calculation agrees well with our first estimate, indicating that, at least at 100km/hour, most of the energy used by the car goes into accelerating the air it is moving through. This is bad news as we can't do much to engineer our way past $\frac{1}{2}\rho Av^2$ — to reduce our energy use we must either drive slower, or less far, or design cars in a completely different shape. However, there is hope for the $\frac{1}{4}$ efficiency of the engine: electric cars are far more efficient, turning about 90% of their electrical energy into mechanical energy. A sustainable future is likely to involve electric cars!

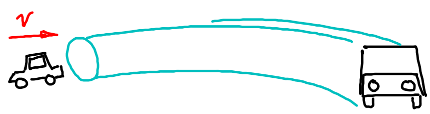

Figure 6: A car moving at a speed v creates behind it a tube of swirling air. The cross-section of the tube is close to frontal area of the car, and the speed of the swirling is close to v

7.2 Planes

People also like to fly long distances occasionally. How much energy does this require? Let us imagine our hypothetical person likes to take one return long-haul flight a year. A long haul flight might be London-Hong-Kong, a distance of about 6000km. While a plane travels, it must also push air out of the way, just as the car did, expending an energy per meter of $\frac{1}{2}\rho Av^2$. To get from this to an estimate of how much energy this might take, we first need to estimate the following easier quantities:

Exercise 17. *How many people travel on a 747 at once. How fast does a 747 travel? What is the cross-sectional area of a 747? Putting these together, what is the energy cost, per person-per-day, of long-haul flying? Remember, the density of air at cruising altitude is about 0.3 of that at ground level.*

This sketch calculation gave, in my case, a rather accurate answer. However, its a bit misleading: in reality aircraft are carefully engineered to have rather less drag than this but must also use energy to generate lift. If we have time, we will go through a better version in class.

8 Wind

Moving air was a problem for cars and planes, but it is also a source of renewable energy. Wind turbines can turn the kinetic energy of wind into electrical energy we can use for useful stuff. How much energy could we get from wind? To get energy from wind we use windmills, better known as wind turbines. These are pairs of blades that spin in the wind, and which in generate electricity from the turning motion. How much energy can a single wind turbine generate?

Figure 7: A wind turbine in Walnut Iowa.

Exercise 18. *What is a "typical" wind speed?*

Exercise 19. *A wind turbine's blades have diameter d. If the wind speed is v and the density of air ρ how much air flows through the turbine in a time t. If the turbine extracts all the kinetic energy from this air, what is the power of the turbine. What is this in watts if the turbine has a diameter of 25m and the wind speed is typical?*

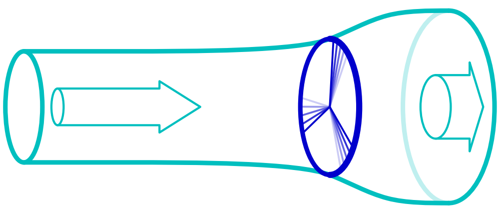

Figure 8: A wind turbine slows wind down and splays it out.

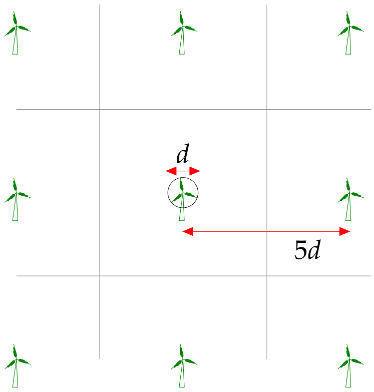

Figure 9: Wind farm layout

Of course, a wind turbine cannot actually extract all the kinetic energy from the air as the air still needs to leave the turbine, as shown in fig. 8. In reality wind turbines only harvest about half the available power. Furthermore, we can't run a civilization on one wind-turbine, we need to cover large areas of land with wind-turbines. So, to work out how much wind power is really available, we need to ask how closely can we pack wind turbines. Clearly we can't put one wind turbine directly behind another one as the air behind a turbine is rather still, but if we put them too far apart we will be wasting valuable wind. So what is our optimal spacing? This is a very difficult question, but we can tackle it using an excellent estimation technique: dimensional analysis. The only quantities the answer can depend on are the diameter of the turbine, d, which has dimensions of length, the wind speed, v which has dimensions of length/time, and the density of air, ρ which has dimensions mass/length^3. The only way we can combine these into something with dimensions of length is to simple say the optimal spacing $l \sim d$. This makes intuitive sense — large turbines need to be further apart. The constant of proportionality cannot be gained by dimensional analysis, but is approximately 5, so $l = 5d$. A wind farm layout is shown in fig. 9.

Exercise 20. *What is the power per-unit-area of a wind farm, if the turbines are on a square grid with spacing $l = 5d$? What does this work out to for a typical wind speed?*

Exercise 21. *Finally, to work out how much wind energy is available to each person, we need to ask what area of wind-farm we can reasonably generate for each person. A first guess would be to cover the entire country in wind-farm, so someone living in HK would have 400 m^2 of wind farm, and each American citizen 32000 m^2. How much wind power per day is this for our table? Clearly we cannot cover the entire world in wind-farms. What would a more realistic estimate of the area we can devote to wind farms be? How much energy per person per day is this? Put this value in the table at the start of the workbook.*

9 Lighting

Another energy need is lighting. How much lighting energy does the average person with a western lifestyle need? Until fairly recently we lit our lives with incandescent light-bulbs. These amount, quite simply, to a thin tungsten wire that we pass a large current through. The current heats the wire to a high temperature (but less than the melting temperature of Tungsten, 3,422°C), causing the wire to emit light a black-body like spectrum of light, as discussed in section 4.

Figure 10: Different types of light-bulb. Modern lights are much much more efficient at turning electricity into light rather than heat.

Exercise 22. *Use the graph of the black-body spectrum (fig. 2) to estimate what fraction of the radiation produced by a tungsten light-bulb working at 3000K is visible light? What is the efficiency of the bulb? (Visible light is 700-400nm).*

Incandescent lightbulbs are clearly grossly inefficient: they emit almost all their power as infra-red, which may heat our homes but it doesn't light them up. Much progress has been made recently in making more efficient types of light. Florescent tubes work by electrically simulating mercury to emit UV light, which in turn strikes a phosphor coating to emit visible light. This is a very different process to heat-driven thermal emission and can achieve 20% efficiency. LED light-bulbs can, at least in principle, do much better again. However, no light bulb can do better than turning all the power it consumes into light. How much light do we actually need? Well, the key insight here is that an old 100W incandescent light bulb used to light a room very nicely.

Exercise 23. *Estimate how much light energy we need per person per day. Put this into the table at the start of the workbook.*

Exercise 24. *Do we need to worry about street lights and traffic lights in the above estimate?*

10 Hydroelectric

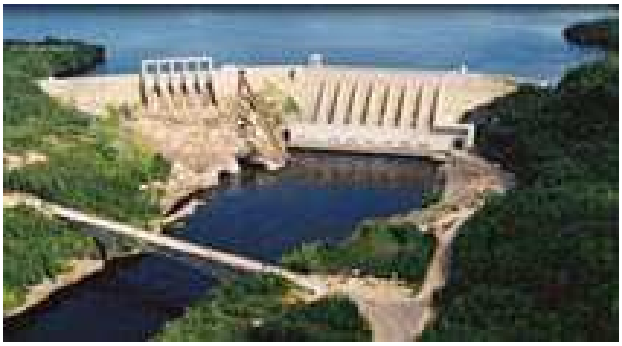

Figure 11: A hydroelectric dam in Quebec.

Rain also supplies us with energy if it is kind enough to deposit water on the top of hills so that we can extract gravitational potential energy from it as it flows down to the sea. This form of energy is called hydro-electric. To estimate how much hydroelectric energy is available we must estimate how much rain falls and how high it lands.

Exercise 25. *Estimate how much rain falls on a square meter of land on a typical day?*

Exercise 26. *How high above sea level does most rain fall? See the height map of the world in fig. 12 for help!*

Exercise 27. *Put your previous two answers together to work out how much hydroelectric energy per day can be attributed to each square meter of land. How much of this might we realistically ever capture? Fill in the hydroelectric Joules per day in the table at the front of the book.*

11 Wave

A close relative of wind energy is wave energy. From the outset it's likely that collecting wave energy will be a bad idea: sustainable energy comes from the Sun, the Sun drives the wind, and the wind drives the waves. Wave energy is thus third hand before we even try to collect it. Waves are created by the wind blowing over the surface of the sea. The wave crests move in the direction of the wind that created them and at about the same speed (that is, at something like 10m/s), but as the wind continues, adding power to the waves, the waves get bigger in amplitude and store more energy. The waves continue to travel long after the wind that caused them dies down, and they loose little power: friction (viscosity) is negligible, a typical wave would go round the world three times before loosing 10% of its

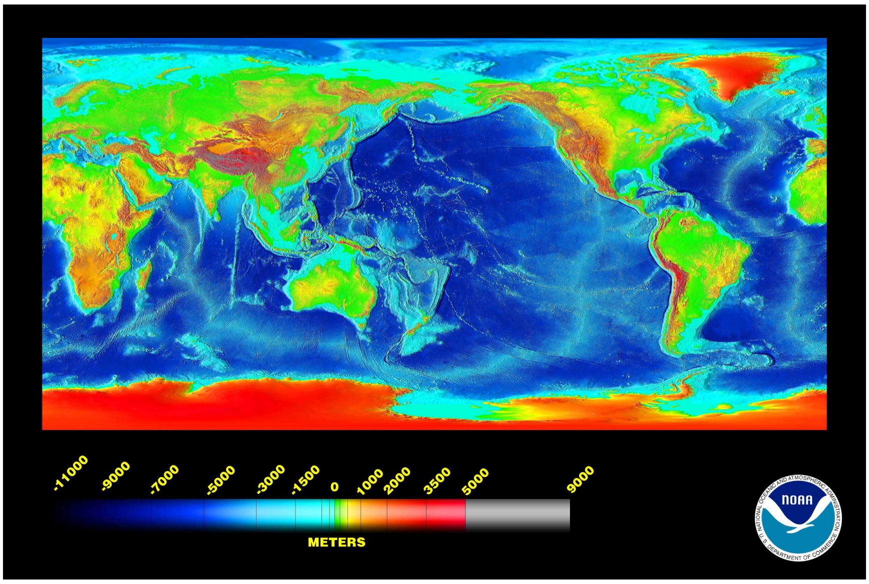

Figure 12: A height map of the world.

Figure 13: A wave harvesting device.

Figure 14: Atlantic waves

amplitude. However, if the waves encounter an energy-dissipating object, such as a sandy beach or a wave power generator, they do indeed give up their energy and down-wind of said object the sea is comparatively calm. Therefore, even with a big ocean, we can collect all the wave energy at the edges (that is at the coast) so we need to ask how much wave energy arrives at each meter of coast: unlike the other forms of energy in this course, wave energy is an power per unit length not a power per unit area.

An important misconception to clear-up about sea-waves is that, although the wave itself can move rather quickly, it does not transport water at the speed of the wave. If you observed seaweed, you will see waves picking it up and putting it down, but it is not carried along at the wave speed. The key picture to have in your head is one like fig. 15 in which the wave-form moves forward, but the packets of water are just sloshing up and down and backwards and forwards without making any progress. This is much like waves on a string: the wave moves forward but it does not carry the string.

Exercise 28. *Estimate the time period T between waves arriving on a beach. Estimate their amplitude h. Using our insight that the wave velocity is about wind-speed, what is the wavelength of the wave?*

Exercise 29. *Estimate the gravitational potential energy per second arriving at a meter of coast. And how about per day?*

Exercise 30. *Estimate how much coast HK, the USA, China and the UK have. How much wave energy is there per person? Fill in the table at the start of the book.*

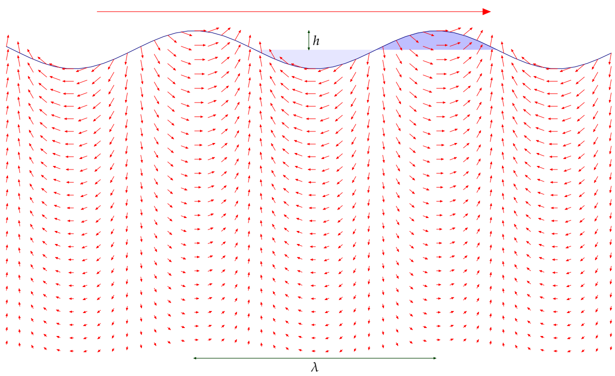

Figure 15: Diagram of a deep water wave. The top arrow indicates the speed of the wave, which is commiserate with the speed of the wind that generated it.

A better approach to estimating the power of the waves would use a little wave physics. This is rather beyond the scope of this course, but we can try to guess the relationship between a waves speed, time-period, wavelength and amplitude so that we can calculate the speed rather than guessing it as we did above. The key insight is that, even if the amplitude of the waves is very small they should still have a finite speed and time period, and ocean waves, being small in amplitude relative to both the depth of the ocean and their wavelength, will travel at this zero-amplitude speed. Having realized this, the only quantities we have to play with to predict the speed are ρ, g and T, so on dimensional grounds we must have $v \sim gT$. If you do the physics properly, it turns out the constant of proportionality is $v = gT/(2\pi)$. We would then guess $v = 16$m/s rather than our above 10m/s, and consequently $\lambda = 160$m.

12 Heating and Cooling

Anyone who has paid a household heating bill knows we use lots of energy for heating and cooling. But how much? We actually heat and cool lots of different things, so we will break out energy usage down into various different categories, and fill out the following sub-table.

Energy Need	**Amount per person per day**
Hot Water	
Cooking	
Hot air	
Cold air	

12.1 Heating water

A good hot bath is a hallmark of civilization.

Exercise 31. *Estimate how much energy is required for a hot bath? How about a shower? Are there any other major uses if hot water we should estimate? Put a figure for hot water in the table above.*

12.2 Cooking and Cleaning

We all prefer hot food, and our ovens, stoves, microwaves and kettles all need to be powered. How much energy does this require? There are two ways to estimate this. One is to look at the electrical power rating of our various devices and estimate how long each of them is on for each day. The other is to ask how much energy it actually takes to heat up our food.

Exercise 32. *If a typical person eats 2kg of hot food and 1l of hot drinks, how much energy must their cooking devices provide every day? How much do we estimate they really provide? Fill in the table above.*

To work out an answer for cooking from an electrical power perspective, the key thing to know is that the power of an appliance is given by $P = VI$, where V is the voltage and I is the current. In Hong-Kong the voltage is $230V$ and the maximum current that can flow is $13A$, so the maximum electrical power available in one socket is about 3kW.

Exercise 33. *What is a typical power rating of a microwave, a kettle, an electric hob and an oven. How many minutes do you think each is in use every day? What is our cooking energy use per day? How does this compare with your previous estimate.*

Exercise 34. *What is a typical power rating of a washing machine, a dishwasher , and a tumble-dryer?. How many minutes do you think each is in use every day? What is our cooking energy use per day? How does this compare with your previous estimate.*

Add up all our cooking and clueing estimates and put them in the above table.

12.3 Hot air

We also like our houses to be toasty warm. In cold countries in winter, this means we have to heat up lots of air. Estimating exactly how much energy this requires is tricky as it is a question of insulation. If our house was perfectly insulated we could just heat it once to the temperature we like then never heat it again! However, houses are not perfectly insulated, so the heat escapes into the cold outside air and we have to provide more heat. Again, we will make two estimates of how much heating energy we need, one based on our knowledge of electric heaters, and one based on the physics of conductivity.

We will imagine that our "typical" person lives in a country where they need to heat their house for half the year to a temperature about 10°C warmer than the outside air. That's pretty typical in the UK, where we like houses that are around 20°C but outside is often 10°C, but of course this is very climate dependent. If you live in a very cold country you might be heating your house to 30° above the temperature outside!

Exercise 35. *What is the power of a small electric heater? How much space can you keep warm with one? Use this to estimate how much energy each of us needs to warm are surroundings each day.*

A more first principals like approach is to consider where all this energy is going. The simple answer is that, after we take precautions to insulate our house from drafts, most of this heat is lots through the walls by conduction. How much heat is lost through an insulating layer depends on its conductivity which, for brick, is about 0.7 $\mathrm{Wm^{-1}K^{-1}}$.

Exercise 36. *If I have a layer of brick of area A and thickness t which is temperature T_1 on the inside and T_2 on the outside, use common sense and dimensional analysis to estimate how many Joules of pass through the brick layer every second. Estimate A, T_1 and T_2 for a modest house in winter. How much heat is lost in a day? How much energy do we need per-person per day to keep the house warm?*

How well do your two estimates of heat energy needs agree? Put a realistic figure in the table at the top of the section.

12.4 Cold air

Of course not everyone lives in cold countries. Many people live in hot countries, and they need to use energy not to heat their air but to cool it down. This is where air-conditioning comes in. In fact the economics of cooling are pretty similar to heating, hot countries might be 30-40C, so we again want 10-20C of temperature change. This will cause heat to flow into the house in exactly the same way that heating the house caused heat to flow out of it, so, just as in heating we needed to supply around 240MJ of energy per house per day, here we need to remove around 240MJ of energy per house per day. However, air-conditioning is actually rather more effective than heating as we aren't supplying the energy we are taking out of the house, we are just moving it from inside to outside. In fact, air-conditioners move about 3 times as much energy as we supply them with, so a 1kW air-conditioner will pump 3kW of heat out of a house. So we might estimate that our ac requires something like 80MJ/day when it runs, which, assuming it runs for half the year and two people share the house, is 20MJ per person per day.

Of course most people only really need heating in the winter of cooling in the summer, so we should probably only include one of the above in our count for a "typical" person.

Exercise 37. *Describe how an air-conditioning unit, or for that matter a fridge, actually works. How do we make things colder than their surroundings?*

12.5 Heat pumps

An air-conditioning unit pumps heat from a cold house to a hot outside, thereby making the house colder and the outside even hotter. A 1kW ac unit delivers 3kW of cooling, so it must actually dump 4kW of heat into the outside. This gives us an idea: can we turn our ac around so it cools the outside and heats the house? The answer is yes, this is called a heat-pump - rather than use our energy to directly heat our house, we use it to pump energy in from the cold outside. We might then expect to get 4kW of house heating for each 1kW of electric energy we supply. The physics and limits of these systems is beyond the scope of this course, but suffice to say there are good grounds for thinking this is a very good idea, and that we might in the future reduce our heating energy needs by around a factor of four. Regrettably systems like this work best in places like the UK where the outside temperature is not much below room temperature. When the temperature gap is very high, which is also when heating consumes the most energy, their performance falls to not much better than standard heating.

13 Tide

Unlike waves, which are third hand solar energy, the tides are driven by gravity, by a combination of the moon and the sun. The basic physics of tides is that the gravitational attraction of the moon falls as $1/r^2$ with distance from the moon, so the ocean on the side of the Earth facing the moon feels a stronger pull towards the moon and hence rises up a little forming a tide. Less intuitively, the ocean at the far side of the earth feels a smaller pull than the Earth itself, so the Earth is drawn slightly closer to the Moon than the ocean, forming a second tide on the far side of the Earth. The Earth rotates once a day, in which time the moon is more-or-less stationary, so we each see two tides a day, once when we are closest to the moon and once when we are furthest away.

Tides are a very attractive form of sustainable energy, principally because, unlike wind, waves, rain and sun, they are genuinely reliable. We can look up the heights and times of tides years in advance. But how much energy might we reasonably harvest from the tides? The basic way we gather tidal energy is via a tide-pool, sketched in fig. 16. The idea is we have a large pool which we fill with water at high tide. We then plug the pool and wait till low tide. At low tide, we unplug the pool and the water flows back into the sea, and we run it past turbines in the process to create electricity.

Exercise 38. *Tidal range around continents tend to be about 4m. How much gravitational potential energy does a tide-pool of area A contain when full at low tide? How much energy can we make per unit area per day?*

The real problem with tidal power is that we need somewhere to put our tidal pond. Typically this is only economically viable using a natural tidal estuary that we can plug with a relatively small dam. In fig. 17 you see a map of the UK with the only real economic opportunity for a tidal garage shown in blue - in the Severn Estuary.

Exercise 39. *Estimate the size of the Severn Estuary? How much energy might it produce per person per day for the UK. Assuming this is a typical amount of extractable tidal power from this amount of coast, estimate how much tidal power might be available per person per day in the other countries we are interested in. Put these estimates in the balance sheet.*

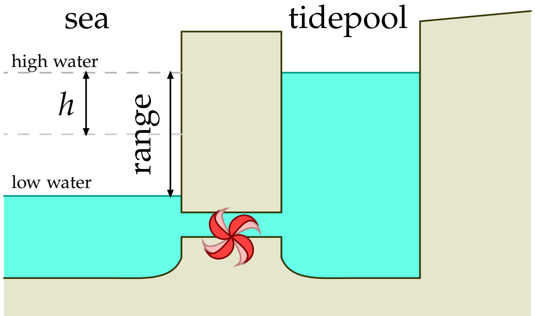

Figure 16: Cross-section of a tide pool. The pool was filled at high tide and now its low tide so we let the water out and generate electricity.

14 Using our gadgets

We continuously here in the news that it is important to turn off our gadgets when we are not using them. But how much energy do we actually use powering our gadgets? To estimate this, we simply need to look up the power rating of a few of them.

Exercise 40. *Use the internet to find the power ratings for a computer, a TV, a radio, a sound system and a bedside clock. How much energy per-person per day do our gadgets need?*

15 Geothermal

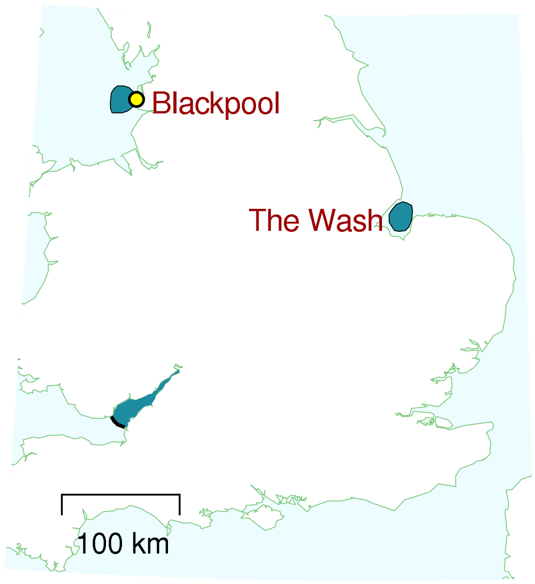

Figure 17: Map of the UK showing the Severn Estuary (lower left) in dark blue.

As anyone familiar with volcanoes knows, the Earth's interior is very hot. If we dig a mine (and the South-African's have diamond mines that are almost 4km deep) we discover that, at least at the surface, the temperature of the earth rises by about 1°C every 100m. Where does geothermal energy actually come from? There are actually two answers to this. When the Earth was first formed it was red hot, and it is still cooling down. Much of the heat in the Earth is residual from its formation. Additionally, in the thin layer of crust on the surface of the Earth there are actually many radioactive elements that decay producing heat. If you look at how much heat escapes from the interior of the Earth to outer-space, these two sources of heat contribute approximately equally.

This is a good moment for an aside on the interior structure of the Earth, shown in fig. 18. The Earths radius is about 6300km. We normally think of it as a solid rock, but in-fact the layer of cool solid rock we live on, called the Earth's crust, is only a few kilometers thick. Beneath the crust lies the mantle. Contrary to popular belief, the mantle is also formed of solid rock (we know this because transverse seismic waves propagate through it), if we had a lump in the classroom, we would all agree it is a solid. However over very long periods of time it flows via a process called solid-state-creep and behaves like a liquid, which is why the Earth is a sphere. Beyond the mantle is the core, which is made of metal (mostly iron). It is divided into the outer core, which is genuinely liquid, and the inner core which, because it is under such high pressure, is again solid.

So, how hot is the middle of the Earth. A naive answer would be to extrapolate the gradient at the surface (1°C every 100m) to the middle to get 63000K, or rather hotter than the surface of the Sun. However, this is complete rubbish. The reason is that, while in solids such as the crust heat moves via conduction, in liquids it moves by convection. Convection is a much more effective way of moving heat than conduction. Think of a pan of hot water: there is very little temperature difference between the top of the water and the bottom of the water, whereas if we replaced the water by a conducting solid the top would probably be cold to the touch. The less conducting the solid, the cooler the top would be and the higher the temperature gradient in the material would be. Similarly, the convecting parts of the Earth (which includes all the mantle except a thin layer beneath the crust) have low temperature gradients and the temperature of the middle of the earth is about 5000K. However, the crust, which is made of solid rocks, which have very low thermal conductivity (they are great insulators) has a very high thermal gradient. This is very handy from the perspective of geothermal energy: the temperature gradient of the Earth is only very high near the surface, so at the surface we don't have to dig very far to get to very hot rocks.

So, can we harvest geothermal energy to power our civilization? Yes, we can. The most common proposed way to do this is as follows: we drill two long parallel pipes into the Earth, as sketched in fig. 20, pump cold water into one, through some deep hot rocks and then back out up the other, giving us a nice supply of steam which we can use to turn turbines and make electricity. However, geothermal energy runs into a real problem: rocks are great insulators, so once we have run our system for a while the rocks at the bottom become cold and it takes a very long time for them to heat back up again. The only way around this is to extract heat from the rocks only at the rate at which heat is naturally flowing through them, so that the rocks themselves do not cool down.

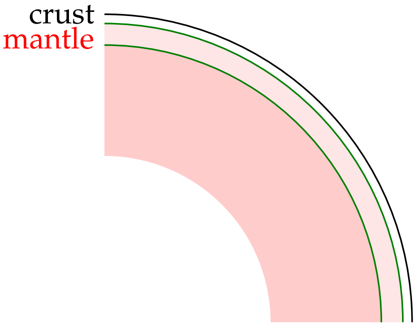

Figure 18: Cross-section of the Earth showing the thin crust on the outside, then a thin layer of solid mantle, then a much thicker layer of deep mantle which, for our purposes, behaves as a convecting liquid.

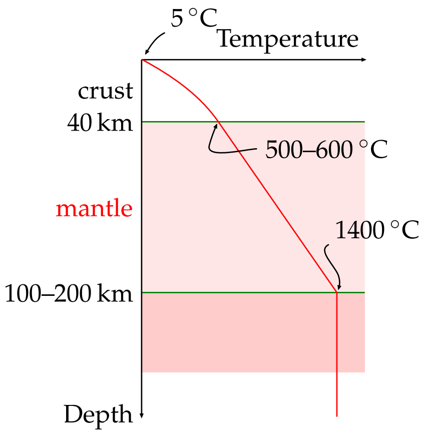

Figure 19: Temperature profile of the Earth's outer layers. The temperature rises almost linearly in the solid conducting section, then is essentially constant in the liquid-like convecting deep mantle.

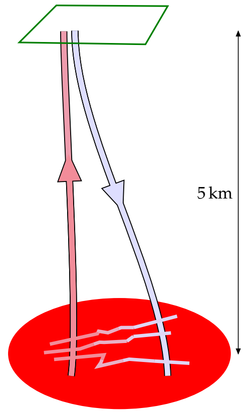

Figure 20: Harvesting geothermal energy by pumping cold water down one pipe then recovering it as steam via another pipe.

Exercise 41. *In fig. 19 you see the temperature gradient of the Earth's crust and the top of the mantle. Assuming rocks have thermal conductivity similar to bricks (about 1 $Wm^{-1}K^{-1}$) how much heat is naturally flowing out of one square meter of the Earth's surface. Even if we catch it all, how much geothermal energy can we provide per-person per day in our countries of interest? Fill these numbers in to the balance sheet.*

Hopefully your answer convinced you that geothermal energy can only make a very small contribution to our sustainable energy future. Of course, we will never even get this much energy: digging deep pipes is difficult and expensive. However, as with hydroelectric power, if you happen to live somewhere where there are few people and high volcanic activity so there are lots of hot rocks near the surface, geothermal energy can make a lot of sense. Iceland is such a place, and they manage to produce 90MJ of geothermal electricity per person per day. This is not to be sniffed at, but even under such optimal circumstances, its still not nearly enough to meet all Iceland's energy needs.

Another approach to geothermal energy is to simply treat it as unsustainable: we can take the heat out of hot rocks and then regard it as gone, in the same way as when we take fossil fuels out of the ground they are gone. This may not be sustainable, but it is likely much better for the Earth's climate as it does not release CO_2 but, since it is unsustainable and technically very challenging, we will not consider it further here.

Material	MJ of Energy to produce 1kg
brick	3
steel	21
plastic	100
aluminium	150
rubber	150
wood	18
paper	36
cotton	55
nylon	250
glass	25
Electronic grade silicon	8000

Table 1: Energy to produce various important materials..

16 Stuff

We are surrounded by stuff: we live in houses, drive cars, drink coke out of cans and eat pizza out of boxes. Making, moving and disposing of this stuff requires energy. How much energy do we need to fuel our stuff habit?

In general, each bit of stuff requires energy at four different times. Consider, for example, a hair-dryer. Firstly, its raw materials — iron, copper, oil – must be extracted and processed into basic materials. Secondly these materials must be assembled into a hair-dryer in a hair-dryer factory. Thirdly the hairdryer gets used to dry hair, requiring energy. Finally, it must be disposed of. A good start in estimating the energy required to sustain our stuff is to look at the production cost of some common materials, shown in table 1. The energy a factory uses to reprocess these raw materials into useful products is rathe hard to estimate, but a good place to start is that simple manufacturing techniques probably still require the material to be completely reworked (e.g. melted again to be cast into a useful shape) and waste some fraction of the input materials, leading approximately to a doubling of the raw material energy. Highly advanced manufacturing such as that required to make PCS can increase the energy required by a factor of 10.

16.1 Little and often stuff

Each day, we each use and dispose of a number of small objects. Exactly what these are of course vary from person to person, but lets imagine a hypothetical person who, on a given day, consumes 3 cans of soft drink, a newspaper and a days worth of food packaging.

Exercise 42. *Estimate the energy cost per day of our hypothetical persons little stuff habit.*

16.2 Big but occasional stuff

Occasionally we also like to buy big things that require huge amounts of energy to make. The biggest two purchases most people make are a car and a house...

Exercise 43. *Estimate the raw material energy cost of a car and a house. If the car lasts for 15 years and the house lasts for 100 years, how much energy do we each need a day if we want cars and houses?*

16.3 Intermediate stuff

Exercise 44. *A typical house also has a fridge, a freezer, an oven, a computer, a tv, a bunch of furniture and lots of small bits and pieces. How long do these last? Which of these are likely to be major uses of energy? Estimate the energy cost of our intermediate size stuff.*

16.4 Moving stuff

As well as being made, our stuff also has to be transported, first during its manufacture and then to us at the end. It turns out that moving things via huge cargo ships is rather energy efficient, so the dominant energy cost of transport is typically the shorter land transport elements. It requires more or less the same energy to send two lorries one mile on one lorry two miles, so the basic unit of measurement for transport costs is the tonne-km: one tone of stuff transported one kilometer..

Exercise 45. *Look back at our previous stuff calculations. What weight of stuff are we using on average each day? How far has it come by road? How many road tonne-miles do we each need every day?*

The energy cost of road-freight is about 3.6MJ per tonne-km. I'm a bit mystified about why it is this high: lorries must be rather inefficient. However, taking this figure...

Exercise 46. *How much energy do we need for our stuff moving needs per person per day?*

17 Solar

Most of the sources of energy we have considered previously are derived from the Sun. For example, the sun heats the air, causing wind, which in turn causes waves. Rather than trying to capture solar energy second or third hand, wouldn't it be simpler to capture it directly?

17.1 How much solar energy is there

In section. 5 we showed that the Sun delivers a power of 1300W/m^2 to the Earth. This looks very promising: it is a far higher energy per unit area than we saw for wind, wave or geothermal. However, we first have to make a few disappointing reductions. Firstly, 30% of this energy is reflected straight back by clouds, so we never get the opportunity to harvest it. Secondly, this is the power-per-unit area of a patch of land pointing straight at the sun, which is true only at the equator at mid-day. As seen in fig. 21, the Earth presents an circular "'net" of area πR_E^2 to the Sun in order to catch this energy, whereas it has a total surface area of $4\pi R_E^2$. Thus, the average solar flux at the Earth is a quarter what it is at the equator at mid-day. Taking all this into account, we might guess there are actually about 200W/m^2 of solar power arriving at a typical point on the Earth. Of course, not all points are equal. In the UK the high-lattitude means that we always make an angle of about 50° with the equator, and there is cloud cover much of the time, so the power density is more like 100W/m^2. In Hong-Kong 250W/m^2 might be available.

17.2 Thermal solar

The simplest way to harvest solar energy is simply to pump water through black pipes on our rooftops. The pipes then absorb solar energy and heat the water inside them. This has the advantage of being tremendously cheep and rather efficient: up to 50% of the incident solar radiation can be caught. The disadvantage is that it only results in moderately warm water, which we can use for some of our heating and washing needs, but which we can't generate electricity from it, and isn't worth moving it long-distances. However, we do have heating and washing needs for warm water...

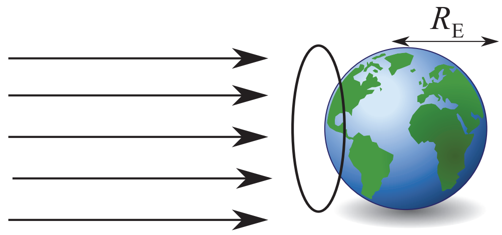

Figure 21: The solar energy hitting the earth (black arrows) delivers 1300W/m^2 of power, but the size of "net" the Earth has to catch this is only of area πR_E^2, while the actual surface of the Earth has area $4\pi R_E^2$.

Exercise 47. *If we all cover our roof-tops with water heating thermal-solar panels, how much energy will we harvest per person per day?*

17.3 Photo-voltaic

We can also use photovoltaic solar cells to directly convert solar energy into electricity. Such solar panels are expensive and intrinsically quantum-mechanical: a photon of light is absorbed by an electron in the material promoting it to a higher voltage. The solar-panel can thus supply electrons at this higher voltage and act as a battery. The jump in energy/voltage the electron makes when it absorbs a photon is a material property, but obviously the absorption will only happen if the photon has at least as much energy as the size of the jump. This gives rise to an optimization problem in designing photo-voltaic cells. The amount of energy we extract from a photon is equal to the jump in energy of the electron, which is less than the energy of the photon. If we choose the energy-jump to be too small then all solar photons have enough energy to be absorbed and cause the jump, but since the jump is small, we are wasting most of the energy. If we choose the jump to be too large then we get lots of energy from those photons which are absorbed, but most photons have too little energy and therefore go straight through the panel. Choosing the optimal wavelength, we are still never likely to get an efficiency of more than around 45%. In reality solar panels typically are about 10% efficient, though very expensive 20% efficient panels do exist.

Exercise 48. *If we all cover our roof-tops with water photovoltaic solar panels, how much energy will we harvest per person per day? Should individuals use thermal or photovoltaic solar panels?*

Exercise 49. *If we start to cover the countryside in solar panels we would gated rather more electricity. Of course, such solar farming requires us to completely industrialize the countryside in question; no more green and pleasant fields. What portion of the countryside might we realistically cover? How much energy would that give per-person per day in our countries of interest?*

17.4 Biofuels

Finally, plants have been busy harvesting solar power for years via photosynthesis. The coal we burn is the result of photosynthesis millions of years ago. We already use plants to provide us with solar power when we eat them, and when we use a wood burning stove. Perhaps we should cover the world in green-plants, then pick them and burn them to produce heat and electricity. The advantage of this would be that plants are cheep (unlike photovoltaic panels), and it will keep the countryside green and pleasant. Of course, we still have the problem that if we grow biofuels on our land we aren't growing food. However, a much bigger problem is that photosynthesis typically less than 1% efficient (at its best it can be 2%, but it's rarely at its best) at turning solar energy into carbohydrate we can burn. That's much less efficient that a thermal-solar panel or a photovoltaic panel. If we try and make electricity out of the plants we are likely to loose another 50% in the conversion process. If we try and make fuel for cars out of them, we again have to process them into nice clean molecules like simple ethanol, which will again probably cost us about half the energy.

Figure 22: An array of solar panels near Santa-Cruz (www.solarwarrior.com). The panels cover 220m^2 and supply an average power of 5kW.

Exercise 50. *If we covered half the country in bio-fuels (we do after-all still need to eat) how much energy would we generate per-person per day?*

Exercise 51. *Of course, we can't cover the country in biofuel and photovoltaics. Nor can we put both thermal and photo-voltaic cells on our roofs. Make a sensible choice for a mixture of solar harvesting devices that can coexist, and put the appropriate figure into the balance sheet.*

18 Food

People, just like cars and planes, require fuel to operate. We call this fuel food. How much energy do we need to feed ourselves?

Exercise 52. *How many calories of food does a healthy adult require each day? What is that in Joules per person per day?*

We may one day discover a way to turn say electric energy into nice food highly efficiently, in which case the above figure would be the whole story. The good news is that, if we eat plants, the energy we eat has been harvested directly from the Sun by the plant, and is already a sustainable energy source. Many people also like to eat animals such as cows and sheep which in turn eat plants. This is rather less efficient as the animal wastes a lot of energy heating itself, moving around, making noises and generally being alive, so rather little of the plant energy an animal consumes ends up as meat energy on a persons plate. How little?

Exercise 53. *How much food energy do I have to give my pigs if I want to eat 200g of pig each day. How about if I want 200g of cow each day?*

Figure 23: Some Miscanthus grass, which is a much talked about biofuel. In the UK miscanthus achieves a power density of 0.75W/m^2

All the above isn't really part of our energy problem: all this energy already ultimately comes from sunlight. We may have a space problem producing all the food we need, but that's a whole different issue. However, the food industry does also use lots of non-sustainable energy fertilizing, gathering processing and transporting our food. The fertilizer energy is hard to estimate, but in Europe, is about 7MJ per person per day. Similarly farm machinery consumes another 4MJ per person per day. We'll already took transportation costs into account in our section on stuff.

Exercise 54. *Choose a representative diet and put an entry for food energy requirements into the balance sheet.*

19 Nuclear

We are familiar with the idea that chemicals contain chemical potential energy, and that if we convert them to different chemicals with lower chemical energy (aka burn them) then we can release some of this energy. Such burning of fossil fuels to release their chemical energy is the basis of our industrialized energy economy. We can also play similar games with atoms. Atoms carry nuclear potential energy, and if we change them from one type to another type, with less nuclear potential energy, we can liberate the difference and use it for our own purposes. This is the essential idea behind nuclear bombs and nuclear power stations. The nuclei of atoms are made out of protons and neutrons (collectively known as nucleons). We can't make or destroy nucleons, but we can take a heavy atom and break it into two smaller ones, as long as the sum of the nucleons in the two smaller atoms is the same as it was in the big one. We can also join two small atoms to make a big one. Curiously, as shown in fig. 24, the atom with the least nuclear potential energy (per nucleon) is iron (Fe^{56}), so we can release energy, either by splitting atoms that are heavier than iron or by joining atoms that are lighter than iron. Only atom splitting (known as fission) has been used successfully to generate electricity, but joining (known as fusion) powers hydrogen bombs and the Sun.

In reality we aren't very good at nuclear reactions, so we can't just take any random atoms and join them together or split them apart in any manner we like. Instead we can only liberate nuclear energy from atoms that are close to spontaneously splitting. The main atom which is both relatively abundant on the Earth and which we can split is Uranium, so almost all existing nuclear reactors use Uranium as a fuel. Uranium actually occurs naturally in two types ^{235}U, which makes up 0.7% of the natural supply and which is easy to split, and ^{238}U which makes up the rest but is rather is not. There are correspondingly two types of nuclear reactor, conventional once-through reactors, which only use the ^{235}U, and fast-breeder reactors which convert the ^{238}U to plutonium-239 and then fission that too. Unsurprisingly the latter obtain about 60 times more energy out of the uranium (since

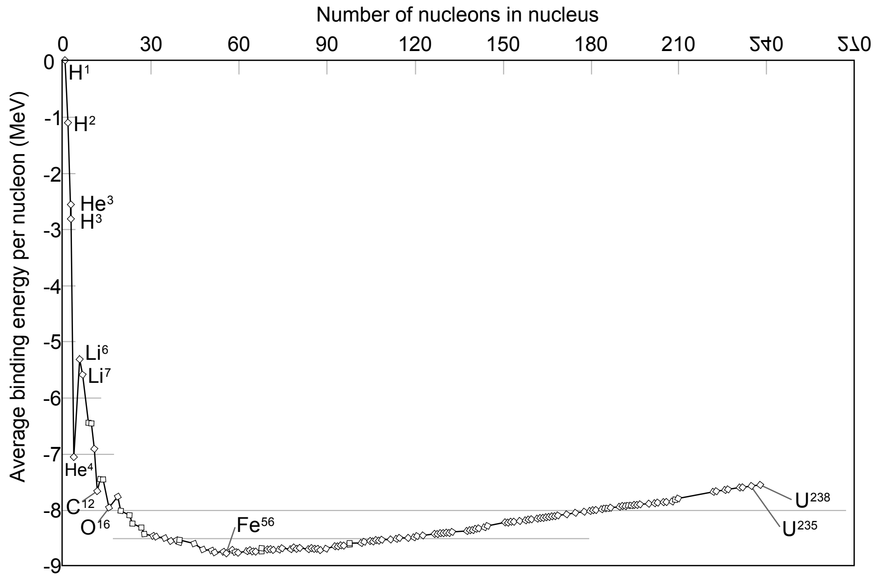

Figure 24: Nuclear binding energy per nucleon for a range of different atoms. Iron (Fe^{56}) has the lowest energy per nucleon, so we can liberate energy either by splitting up atoms heavier than iron or joining together atoms that are lighter than iron.

Source	Millions of tons of Uranium
Conventional uranium ore	4.7
Phosphate deposits	22
Seawater	4500

Table 2: Known world uranium resources.

there is much more ^{238}U in the uranium we dig up).

The good news about nuclear energy is that you get a huge amount more of it out of each kg of Uranium than you would chemical energy out of a kg of coal. Indeed, per atom, there is about a million times more nuclear energy available than chemical energy. A once through reactor gets about 200GJ of energy from each kg of uranium, a fast-breeder gets about 12TJ. The bad news is that Nuclear energy is not a sustainable energy at all. We have a finite supply of uranium in the Earth, just as we have a finite amount of fossil fuel. When we have used it, it is gone. We shouldn't really be discussing it in this course at all. However, it is a low-carbon energy source, so it might have a role to play it avoiding climate change, even if we can't rely on it for ever, and just because it is technically unsustainable doesn't necessarily mean we are actually in danger of running out. So how much uranium do we actually have at our disposal? Table. 2 lists the known reserves, distinguishing three types. The first are conventional ores of the type currently mined. The second are phosphate ores which are not currently economic to mine, but which have been historically and which are certainly available for use if we want them. The third is uranium dissolved in sea-water.

Exercise 55. *If we use our Uranium ore deposits in a semi-sustainable way, by spreading them over 1000 years, how much energy can we produce per-person per day from the known reserves using once-through and fast-breeder reactors?*

Whether this is really accessible is a somewhat open question: no-one has ever extracted such uranium commercially. Small amounts have been extracted on an experimental basis and, though the costs are about \$150/kg, rather than the current commercial cost of \$20/kg, these are actually pretty negligible compared to the other costs of nuclear power stations. (Interesting fact: the fuel in a nuclear power station is a small fraction of the total cost!). However, even with a utopian world view, we can't hope to extract it all immediately since most of the sea-water is at the bottom of the ocean where we can't hope to get to it. The oceans actually circulate rather slowly: the water at the bottom of the oceans only circulates to the top then back to the bottom every 1600years.

Exercise 56. *Very optimistically we might extract 10% of the oceanic uranium in each oceanic cycle. How much energy would this provide per-person per-day in once-through and fast-breeder reactors?*

Finally, in principal we can also acquire nuclear energy by fusion of small atoms rather than fission of big ones. This is a tried and tested bomb technology, and how the Sun is powered, but no-one has successfully made it work as a electrical power station. Scientists are working on two flavors of fusion reactors, DT reactors which fuse deuterium and tritium to make helium, and DD reactors which fuse deuterium and deuterium. The nice thing about the DT reaction, which is the main focus of modern fusion research (a project called ITER), is that it "only" requires a temperature of 100000000K to get going whereas the DD reaction requires 300000000K. The trouble with the DT reaction is that we don't really have any tritium on earth, so we have to make it out of lithium, which is a finite resource.

Exercise 57. *There are estimated to be about 9.5 million tons of lithium ore deposits on Earth. We might hope DT fusion would deliver 8GJ per gram of lithium. If we use it all over 1000 years, how much is this per person per day? The total oceanic resources of lithium are estimated to be* 2.3×10^{11} *tons. Using the same assumptions as for extracting oceanic uranium, how much energy per person per day could this lithium deliver?*

Exercise 58. *If we get DD fusion to work, the good news is that fusing 1g of deuterium produces a mind-boggling 360GJ of energy, and there is 33g of deuterium in every ton of water. Using the same assumptions as for extracting oceanic uranium, how much energy per person per day could this deuterium deliver?*

Exercise 59. *Put some realistic numbers for nuclear power into the balance sheet.*

The moral of this final tail is that, although fusion remains far too speculative for us to rely on, continuing to research funding for fusion must be a good idea.

20 Making a plan that adds up

We have now finished our survey of our major energy needs and energy sources. Take a look at our balance sheet, and add the two sides up? Do they balance? Can we live off sustainable energy? What compromises will we have to make to do so?

Exercise 60. *Think about cutting energy needs and boosting energy supply to design an energy plan for China that balances. Is there any such plan you are happy with?*

I hope this final exercise brings out the three major lessons from this course. Firstly, renewable energy sources are very diffuse. If we are to harvest meaningful amounts of energy from them, we are going to have to cover transform our countryside, covering large portions of it with solar panels, wind turbines and the like. This is unlikely to be popular. Secondly, even if we do this, it is going to be a struggle for most countries to supply their energy needs. It's clear a city like HK cannot, and nor even can densely populated country like the UK. China has more of a chance while the USA, with its low population density and strong sunshine, clearly could. We are therefore also going to have to invest in infrastructure to move renewable energy from where it is abundant to where it is needed; the UK, for example, should be thinking about solar panels in the Sahara. Thirdly, most of our sources of renewable energy produce electricity, so we to make sustainable energy work, we need to electrify our energy uses: we need electric cars and trains for transport, electric heat pumps to heat our houses and electric hobs to heat our food. Finally, although it isn't sustainable, we should think seriously about nuclear power. It arguably offers an indispensable proven short term route to reliable low carbon power, which can act as a medium term stop-gap while we build, test and optimize the rest of our sustainable infrastructure.